Mathematical Economics - Game Theory

Introduction

Game theory is a model of applied mathematics broadly used in many fields ranging from economics to project management. Game theory is a study of how different parties[1] make actions[2] that are interdependent. This interdependence results in both parties needing to consider the opposing parties' decisions before decision-making. Game theory aims to produce an optimal decision-making strategy for interdependent actors in a strategic setting. There are many game theory examples such as the Prisoner's Dilemma, chess, Battle of Sexes or even flipping coins. People use applications of game theory in their daily lives without realizing it. Although there are many applications of game theory, for a game to be able to predict the choices of players the actions of players should align with the assumptions of game theory. This essay explains the concepts, assumptions, real-life applications, and the different types of games of game theory.

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[1]Each individual decision maker in the game trying to maximize his utility.
[2]Any decision in a game

1. Brief History of Game Theory

Game theory was originally developed by Hungarian-born American mathematician John von Neumann and his German-born American economist Princeton University colleague Oskar Morgenstern in the 1940s. Game theory was developed by the two to solve problems in economics. Their book “The Theory of Games and Behavior” proposed a new approach to mathematics. Their theory was later developed by John Nash in the 1950s and then evolved into a branch of applied mathematics. The term, Nash equilibrium, is named after its inventor, John Nash (Britannica).

2. Key Concepts in Game Theory

2.1. Utility

Utility in economics refers to the benefits obtained from consuming an item or service. There are many concepts related to utility such as marginal utility, total utility, and cardinal and ordinal utilities. Total utility refers to the comprehensive benefit gained by obtaining a certain amount of service or item, whereas marginal utility refers to the additional utility gained by each additional service or item. The graph of the marginal utility function can be represented as shown in the figure below.

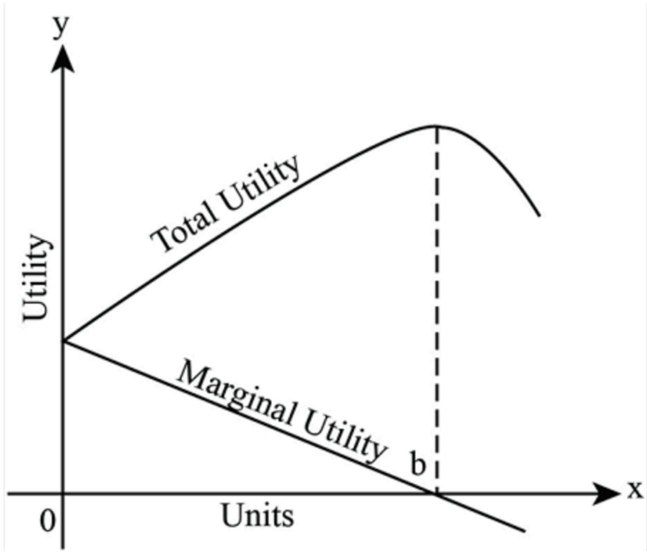

Figure 1 - Total Utility and Marginal Utility as the units of the item consumed increases.

The theory of marginal utility was then further developed by Daniel Kahneman and Amos Tversky in 1979 leading to the formation of the Prospect Theory[3]. Prospect theory is a psychological theory that aims to explain the decisions of people, especially

[3] Prospect theory describes how people perceive and make decisions when presented with alternatives that involve risk, probability, and uncertainty. It is also referred to as the loss aversion theory. For further explanation, visit: https://www.press.umich.edu/pdf/0472108670-02.pdf

investors in financial markets, according to the loss-averse nature of people. The theory claims that people are risk-averse when it comes to gains but tend to be risk-seeking when faced with a loss. Figure 2 shows the basics of prospect theory on a graph.

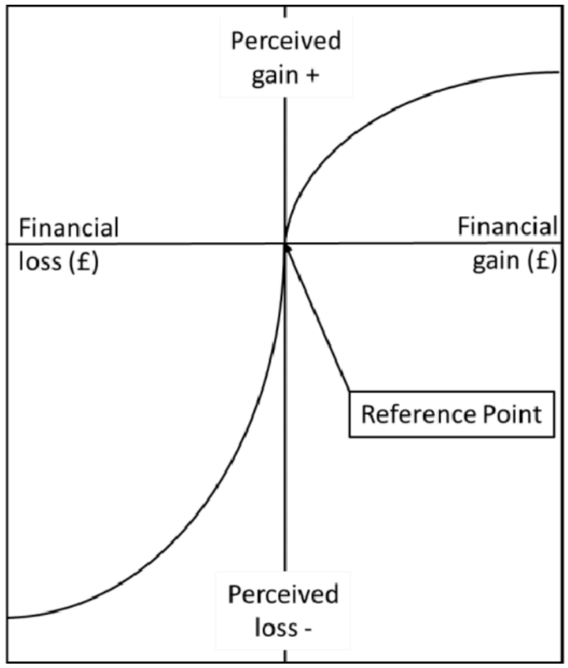

Figure 2 - Prospect Theory

Rational players in game theory are assumed to make interdependent choices to maximize utility while they are making decisions. Utility in game theory is commonly referred to as the "payoff". In the normal form of a game (See 2.4 Definition of a Game in Normal Form), the utility is expressed as the utility function. The utility function is "a quantitative representation of players' preference relations over the possible outcomes of the game" (Maschler et al.). It is used as a quantity to show the preferences of players between two payoff functions, as a way to demonstrate the preference of certain players under uncertain conditions. Each agent has its utility function, and these utility functions are listed on a matrix as quantities.

2.2. Game

A game, according to the game theory, is much different from the activities people refer to as games in real life. Games in game theory serve as a model of players' actions and preferences. Their models help decision makers to predict payoff and detect the optimal strategies. The main properties of a game are its players, players' actions, and corresponding payoffs or utilities of those actions for the players. Actions are all possible choices a player can make in a game, and the payoffs of those actions are the utilities that self-interested agents/players care about. In a game theory, all players are assumed to be self-interested, meaning that their main purpose is to maximize their utility. As a result, while defining a game, one should determine whether the game is finite. A finite game contains a finite number of players with a finite number of actions. Some common examples of such games can be listed as: chess, checkers, tic-tac-toe, and matching pennies. On the other hand, infinite games are harder to handle than finite games, as there are unknown and known players and/or actions in those games. Using the number of players in a game, the simplest way to classify a game is to classify it depending on the number of players (n) in a game by calling it an *n-player game*. Games in each category such as one-person, two-person, and n-person (n is assumed to be greater than 2) have their distinctive features. There are many other ways to classify games, and those will be elaborated on in the upcoming sections (Game Theory).

2.3. Rationality

In game theory, the players are assumed to be rational, which in the context of game theory means that the players will be aiming to maximize their profit/ payoff functions. Game theory assumes that the organs/companies/individuals are self-interested agents who aim to maximize their utility. However, players, especially when the players in question are people, who do not make decisions like a *Homo economicus*[4]. Many philosophical and psychological questions may arise regarding rationality, however, in the context of game theory rationality means that they will take the initiative to increase its utility (Steele and Stefánsson).

2.4. Definition of a Game in Normal Form

Normal form (also referred to as the Matrix form) is used for defining simultaneous action games. In these types of games, the players can't foresee what the opposing player will do unless there is a strictly dominant strategy that a self-interested agent will be expected to choose as long as rationality applies. A classic definition of a game theory in normal form is as follows

The set of players : $N = \{1, 2, 3, 4, \ldots , n\}$

One of the ways to categorize games in game theory is the number of players in a game. Above is the notation for an n-player game, n could be any integer ranging from 1 to infinity. Since a one-player game would simply be an individual decision game and no actions would be interdependent on another player, most of the games that will be discussed in this essay will consist of 2 or more players. In a game, a player i has a set of actions a_i available to the player, which is commonly referred to as pure strategies. The actions the players have selected that give a certain payoff to each player are called the action profile. The number of actions a player can take can be finite or infinite, depending on the game. This is the set of all pure strategies that can be denoted by

$$a = a_1 \times \ldots \times a_n \quad \text{or} \quad a = (a_1 \times \ldots \times a_n)$$

Last but not least, below is the utility function for player i who chose to play action a

$u_i : A \ -> \ R$ therefore, $u_i(a)$ is i's payoff when action a is picked.

[4] a hypothetical person who behaves in exact accordance with their rational self-interest, an individual with infinite ability to make rational decisions.

Aside from these function definitions, games in normal form are most commonly presented on a table. Let's look at an example:

(Player 1, Player 2)		PLAYER 2 (Prisoner 2)	
		Stay Silent	Betray and Testify
PLAYER 1 (Prisoner 1)	Stay Silent	(-1,-1)	(-10,0)
	Betray and Testify	(0,-10)	(-4,-4)

Figure 3 - Prisoner's Dilemma Payoff Matrix

This is an example of the game "Prisoner's Dilemma" represented by a payoff matrix. The prisoner's dilemma consists of 2 players, prisoners arrested and interrogated in separate rooms simultaneously. The prisoners can stay silent and not tell anything about the crime, while their other option is to betray and testify against the other prisoner. If none of the prisoners testify, the burden of proof will not be reached and each prisoner will only be given 1 year of prison. If one of them decides to testify and the other decides to stay silent, the one testifying will be let go while the other prisoner will be sentenced to 10 years in prison. On the other hand, if both testify, both will be sentenced to 4 years in prison. The payoff functions of players can be demonstrated on the payoff matrix, in this case, the numbers represent the utility they will gain as a result of being sentenced to prison, as shown in Figure 3. Each year of prison corresponds to adding -1 to the payoff function. The intersection of the actions that the players take on the matrix gives the payoff function of each player for that action profile.

Each player is represented by a dimension of a matrix, in this case, Prisoner 2 determines the horizontal location of the payoff, and Prisoner 1 determines the vertical location of the payoff. Each choice of Prisoner 1 is specified with a row, and each action of Prisoner 2 is represented by a column of the matrix. The payoff functions corresponding to the resulting action profile for each Prisoner are given on the matrix. In this case, the second number represents Prisoner 2's payoff function and the first number represents Prisoner 1's payoff function (Cruz).

The dominant strategy of a player means that for all action profiles dependent on other players, that strategy that the player takes will always give a greater payoff than that player's other strategies. The term dominant strategy is divided into two, strictly dominant strategies and weakly dominant strategies. Here are the mathematical definitions of strictly and weakly dominant strategies.

Definition of a Strictly Dominant Strategy

s_i **strictly dominates** s'_i if $\forall s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$

Definition of a Weakly Dominant Strategy

s_i **strictly dominates** s'_i if $\forall s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$

A strategy s_i is strictly dominant for all s_{-i} (all other actions of other players) in the set of strategy profiles if s_i has a greater payoff function than all the other strategies for the player taking the strategy profile s_i. Similarly, a strategy profile is weakly dominant if it gives a payoff greater than or equal to the other strategy profiles the player can take.

1\2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

Figure 4 - A 2 Player Game Payoff Matrix

For example, in the matrix in Figure 4, the dominant strategy for player 1 is c as the player gets more utility whatever the other player does. Player 2 does not have a strictly dominant strategy, however, y is his weakly dominant strategy as the utility he gets from y is either greater than or equal to his other actions whatever player 1 does ("Game Theory").

2.5. The Nash Equilibrium

The Nash Equilibrium gets its name from its founder, John Nash. Nash equilibrium is both one of the most essential and the most complex components of game theory. The simplest definition of a Nash equilibrium can be described as the strategy profile for which none of the players in the game would take the initiative to deviate. This term does not mean that the Nash equilibrium is the option with the highest payoff for a player, and should not be confused with Pareto optimality[5]. In other words, in the case of a Nash equilibrium, no other action (a_i) a player can choose can give a better utility function ($u_i(a_i)$) when the other player(s) choose certain actions. Nash equilibrium means that it is the action profile from which no player would deviate. According to John Nash's theorem from the 1950s, every finite game has a Nash equilibrium (Chen).

2.5.1. Pure Strategy Nash Equilibrium

The pure strategy Nash equilibrium is the most fundamental type of Nash equilibrium there is in game theory. In a pure strategy, Nash equilibrium an action of player i a_i has been assigned a positive probability value of 1 and there is only one action that has been assigned a positive probability value (this makes sense because the probability values of all actions a player

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[5] A strategy profile is said to be Pareto optimal if it is not Pareto dominated by another strategy profile, in other words, there isn't a strategy profile that makes all parties involved better off than that strategy profile.

can take should sum to 1). To understand what a pure strategy Nash equilibrium is, one should be familiar with the concepts of best response, dominant strategy, and dominated strategy.

Mathematically, the pure strategy Nash equilibrium of a game is defined as below:

$a = \langle a_i , \dots , a_n \rangle$ is a Nash equilibrium if $\forall_i \, a_i \in BR(a_{-i})$

This equation could be explained as: a is the action profile of player i, and that action profile is a pure strategy Nash equilibrium if, for all a_i, a_i is an n element of the set of best responses (BR). The best response is the action with the greatest utility for players other than i, as a response for an action of a player i. Although all pure strategy Nash equilibrium have some dominant strategies involved, there is a special type of Nash equilibrium that consists of the action profile that allows all players to play their dominant strategies. This is called the dominant pure strategy Nash equilibrium, and it is the strongest and most stable type of Nash equilibrium.

2.5.2. Mixed Strategies

John Nash states that all games have Nash equilibrium. However, he doesn't say that all games have pure strategy Nash equilibrium. Mixed strategy Nash equilibrium is the other type of Nash equilibrium. In mixed strategy Nash equilibrium, instead of assigning the positive probability of 1 to only one action profile, the Nash equilibrium assigns positive probability to multiple action profiles. Each player distributes their probability to do the actions across their support. Support is a term used to refer to actions that have a positive probability assigned to them. In mixed strategy Nash equilibrium, one can not predict the utility of the players by only looking at the matrix of the game in normal form. Instead, to find the utility of a player, the possibility of acting to that action should be considered. While calculating/predicting the mixed strategy Nash equilibrium for a game, the calculations for the expected utility should be done.

Mathematically, the mixed strategy Nash equilibrium of a game is defined as follows

$s = \langle s_i, \ldots, s_n \rangle$ is a Nash equilibrium iff $\forall_i \, s_i \in BR(s_{-i})$

Although, it is quite similar to the pure strategy Nash equilibrium. There is one significant factor that differentiates these two terms. The action profile (a) turns into a strategy profile (s). This change, again, refers to the fact that for the equilibrium to happen, players don't have one single pure strategy action profile that yields an equilibrium. Instead, we calculate the probabilities of each player playing each action for which the other player(s) can randomize. In order for a player to be able to randomize, the expected utility functions for each of its choices should be equal.

2.5.3. Finding/Calculating the Nash Equilibrium

There are many unique ways proposed to find the equilibrium in a game theory. In the case of pure strategy Nash equilibrium, it is fairly simple. As long as the Nash equilibrium is a pure strategy equilibrium, one can look for the dominant strategies and best responses. For instance, for an action profile to be a Nash equilibrium, it needs to be the best response. If one of the players has a dominant strategy, the action profiles of that strategy can be crossed out and the other strategies can be evaluated. Let's look at a simple example of a 2 x 2 game:

(Player 1, Player 2)		PLAYER 2 (Woman)	
		Opera	Football
PLAYER 1 (Man)	Opera	(1,2)	(0,0)
	Football	(0,0)	(2,1)

Figure 5 - Battle of Sexes Payoff Matrix

The given payoff matrix belongs to the game “Battle of Sexes”. This is a cooperative non-zero-sum game (see 7. Types of Game Theories) in which the sides need to cooperate to gain utility. However, the problem arises when two sides have different payoffs for different activities. The woman likes it more to go to the opera than to a football game, so her utility for opera is 2 while it is 1 for football. Meanwhile, for the man, it is the opposite. If they do not choose the same choice, they will be going to different places and since they are separate, and they can’t spend time with each other, they will have no utility. This is a simultaneous move game, so these players will do their actions at the same time, and they do not know the other’s actions. For this game, there isn’t a single pure strategy Nash equilibrium. For this reason, we need to do expected utility calculations for both players. For example, we should distribute probabilities for each action of Player n so that other player(s) can randomize. For a player to be able to randomize, each action of that player, the expected utility functions of those actions should be equal.

Let us say that Player 1’s probability of playing “Opera” is p and the probability of playing football is 1-p. For the 2nd player, these probabilities are q and 1-q respectively.

For player 2 to randomize, the utility function for player 2 of opera and football should be the same.

(Player 1, Player 2)		PLAYER 2		Probabilities
		Opera	Football	
PLAYER 1	Opera	(1,2)	(0,0)	p
	Football	(0,0)	(2,1)	1-p
Probabilities		q	1-q	

Figure 6 - Battle of Sexes Payoff Matrix with Probabilities

Utility function for opera for player 2:

$2p + 0(1-p)$

Utility function for football for player 2:

$0p + 1(1-p)$

For the 2nd Player to randomize:

$0p + 1(1-p) = 2p + 0(1-p)$

$2p = 1-p$

$3p = 1$

$p = \frac{1}{3} \quad 1-p = \frac{2}{3}$

The first players' strategy profile is:

$(\frac{1}{3}, \frac{2}{3})$

This means that the first player will go to opera ⅓ of the time and to football ⅔ of the time.

On the other hand, the same should apply to player 2 as well, for this to happen:

Utility function for opera for player 1:

$1q + 0(1-q)$

Utility function for football for player 1:

$0q + 2(1-q)$

For the 2nd Player to randomize:

$0q + 2(1-q) = 1q + 0(1-q)$

$2-2q = 1q$

$3q = 2$

$1-q = ⅓ \quad q = ⅔$

The second players' strategy profile is:

$(⅔ , ⅓)$

This means that the second player will go to opera ⅔ of the time and to football ⅓ of the time.

This makes the overall mixed strategy Nash equilibrium:

$((⅓ , ⅔),(⅔ , ⅓))$ ("Game Theory").

2.5.4. Hardness Beyond 2 x 2 Games - The Complexity of Finding the Nash Equilibrium

Normally according to John Nash, "Every finite game has a Nash equilibrium." Thus, determining whether there is a Nash equilibrium in a game is not classified as an NP-complete complexity of a problem. However, if one attempts to find some information more than the Nash equilibrium, then it could be classified as a problem with NP-complete complexity. This information can include uniqueness, Pareto optimality, guaranteed payoff, guaranteed social welfare, action inclusion, and action exclusion while finding the Nash equilibrium. Some proofs for von Neumann's 1928 theory, the existence of Equilibrium in 2-player can be listed as Danzig '57 and Khachiyan '79. Many other proofs were emerging after John Nash proved the existence of a Nash equilibrium in all finite games. All of these proofs use Brouwer's fixed point theorem. Some examples of these newer proofs for equilibrium in multiplayer games are Lemke-Howson '64, Porter et al. '04 ("Game Theory").

3. Types of Game Theories

3.1. Cooperative vs. Non-Cooperative Games

3.1.1 Cooperative Games

Definition

In cooperative games, there are two agents with goals that they can reach at the same time, or they aim to cooperate for them to have the maximum benefit. These are the games in which players can win and lose at the same time (Hayes). Most of the time, these games are non-zero-sum games, as there isn't necessarily a loser and a winner for every action profile.

Example

One of the classical examples of coordination games is the driving game. Here is the payoff matrix for this game.

(Player 1, Player 2)		PLAYER 2	
		Left	Right
PLAYER 1	Left	(-100,-100)	(10,10)
	Right	(10,10)	(-100,-100)

Figure 7 - Cooperation Game Payoff Matrix

In this game, if the cars turn in the same direction, they crash, and they have a utility of -100. However, if they go in different directions, they can happily continue their ways, and they would avoid a head-on collision, which gives them 10 utility. They need to cooperate to avoid the action profile for which there is less utility for both of them. Battle of sexes can also be given as examples for this type of game (See Finding/Calculating Nash Equilibrium).

3.1.2. Non-Cooperative Games

Definition

Non-cooperative games are games in which sides have contrasting utility functions. This means that there is a winner and a loser in a game. They don't need to be always zero-sum games but non-cooperative games require players whose utility increases for an action profile which decreases the utility of the opposing player(s).

Examples

An example of a non-cooperative game is the matching pennies game. Its game matrix is as follows:

(Player 1, Player 2)		PLAYER 2	
		Heads	Tails
PLAYER 1	Heads	(-1,1)	(1,-1)
	Tails	(1,-1)	(-1,1)

Figure 8 - Matching Pennies Game Payoff Matrix

If we look at the matrix, we can see that player 2 wins (gets a utility of 1) when the pennies' sides match, whereas player 1 wins when the pennies do not match. This is an example of a zero-sum game as the sum of the utilities for players for an action profile is 0. This means either both players receive a payoff of 0 or one of the players earns negative utility while the other earns a positive utility indicating a winner and a loser, such as in the game above. As the purpose of the players is not to cooperate but to win on their own, this is a non-cooperative game.

3.2. Zero-Sum vs. Non-Zero Sum Games

3.2.1. Zero-Sum Games

Definition

In zero-sum games, there is a direct competition between the players and while some players win, some players must lose. This is because there needs to be a negative utility for each positive utility earned by a player to make the sum 0. These types of games are commonly non-cooperative games (Hayes).

Examples

An example of a zero-sum game would be a sports competition, especially a game between the goalie and kicker.

(Player 1, Player 2)		Goalie	
		Left	Right
Kicker	Left	(-1,1)	(1,-1)
	Right	(1,-1)	(-1,1)

Figure 9 - Goalie Game Payoff Matrix

There has been research done on this game that, ideally, this is not a zero-sum game and that the utility distributions depend on how well the players play or goalies play. For the sake of the assumptions of game theory, we are going to assume that this game is defined with the matrix above. The goalie aims to catch the ball to save the goal, and the kicker aims to score a goal. As a result, when the goalie jumps on one side of the net and the kicker kicks to that side, the goalie catches and wins. If not, the kicker scores the goal and wins. The sum of their utilities for each action profile is zero, thus this game is a zero-sum game ("Game Theory").

3.2.2. Non-Zero Sum Games

Definition

Non-zero sum games are games in which the utility of players for an action profile does not sum to zero. These games can be cooperative as well as non-cooperative (Hayes).

Examples

For example, chess is a non-zero-sum sequential move game with perfect information. Other examples of this game is Cournot competition, TCP Backoff game, deadlock and chicken.

Let's take a look at the TCP Backoff game. This game has a similar structure to the prisoner's dilemma game. Internet traffic is governed by the TCP protocol. When the protocol is correctly implemented, it includes a “backoff mechanism”: if the rates at which a sender sends information packets into the network cause congestion, the sender reduces this rate for a while until the congestion subsides. A defective implementation of TCP does not back off when congestion occurs. Imagine that you and a colleague are the only people using the internet. You each have two possible strategies: C (using a correct implementation) and D (using a defective one). If both you and your colleague adopt C then you will both experience an average packet delay of 1 ms. If you both adopt D you will both experience a delay of 3 ms, because you will both experience more lost packets. If one of you adopts D and the other adopts C then the D adopter will experience no delay at all, and the C adopter will experience a delay of 4 ms. The payoff matrix of this game is as follows:

(Player 1, Player 2)		You	
		Correct	Defect
Your Colleague	Correct	(-1,-1)	(-4,0)
	Defect	(0,-4)	(-3,-3)

Figure 10 - TCP Backoff Game Payoff Matrix

On the matrix, you may see that the sums of utilities of players do not add up to 0. Hence, this game is a non-zero-sum game ("Game Theory").

3.3. Simultaneous Move vs. Sequential Move Games

Not all games happen in an environment where players are required to make their choices at the same time. Some games include time and order while taking action.

In simultaneous move games, agents make their actions at the same time, and they do not know what the other player is going to do before they do the action. These types of games are commonly represented by a payoff matrix which consists of players as the dimensions of the matrix. All the games explained previously, such as the Battle of Sexes and the Prisoner's Dilemma, were simultaneous move games.

On the other hand, there are sequential move games. This type of game allows players to make their decisions at different times, usually one after the other. The Ultimatum game is an example of this type of game. This game is represented in extensive form, which is beyond the scope of this paper. Nevertheless, Figure 11 demonstrates a representation of the Ultimatum game. In these types of games, the players have information about the previous move the other player made. In the Ultimatum game, there are two options for the first player, a fair share or an unfair share. The other player can either accept the proposed share by the first player or reject it. If the second player accepts the proposal, they will both get the share that was specified by player 1, however, if the second player declines the share, neither of the players gain any share.

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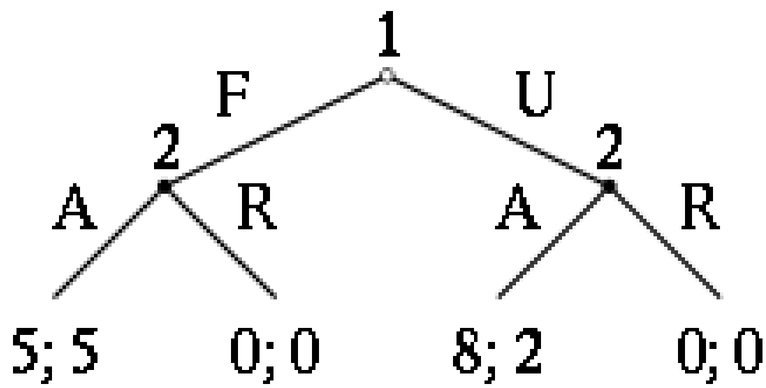

Figure 11 - Ultimatum Game in Extensive Form

As you may see from the form, the choices are time-dependent and the second player has information regarding what the first player did in the previous round ("Ultimatum Game").

Repeated Games

So far, we have only talked about one-shot games. One-shot games are games in which only one round is played. Repeated games, involve playing one game multiple times with the same or different strategies against players with different/random strategies. For example, we know that the optimal strategy for a player in the game Prisoner's Dilemma is to betray and testify. However, this isn't the case for repeated games. If a player adopts a strategy to always cheat, it isn't guaranteed for that player to win the greatest total utility. For example, let us define 3 strategy profiles. The first strategy profile always betrays, the second strategy profile randomizes, and the third one does what its opponent did in the last round. Call these people "Cheaters", "Random" and "Copycats" respectively. With these types of profiles, let's establish a population. These people will play 10 games each round with other players and last five players with the lowest utilities will be eliminated while the first five players with the most utilities will be duplicated. If we establish a population with a nearly equal number of each type of player, we will see as a result of running the simulation that the “Copycats” will dominate the population.

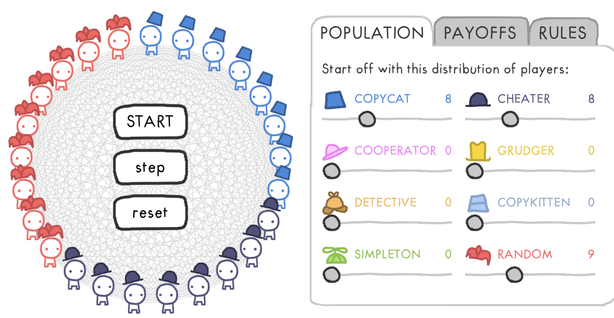

Figure 12 - Prisoner's Dilemma Simulation Starting Screen

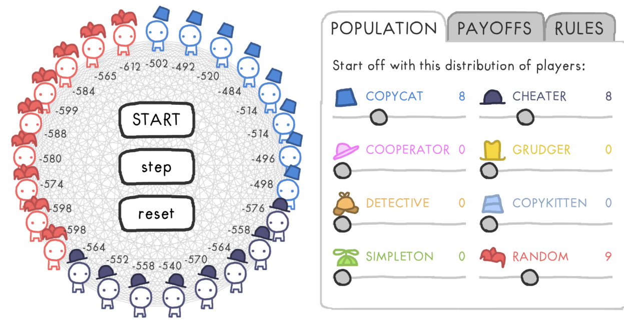

Figure 13 - Prisoner's Dilemma Simulation Elimination 1

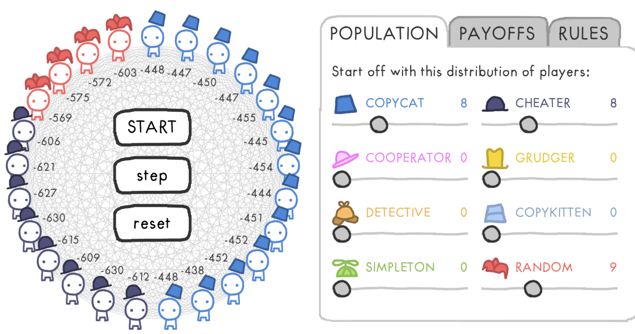

Figure 14 - Prisoner's Dilemma Simulation Elimination 2

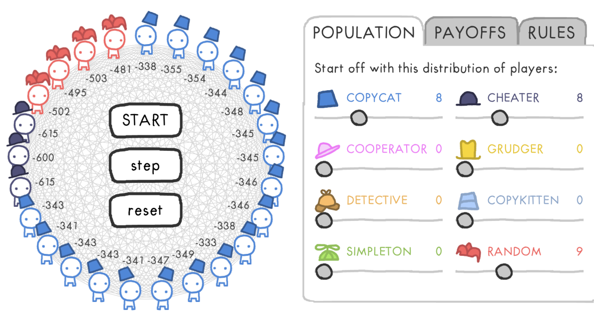

Figure 15 - Prisoner's Dilemma Simulation Elimination 3

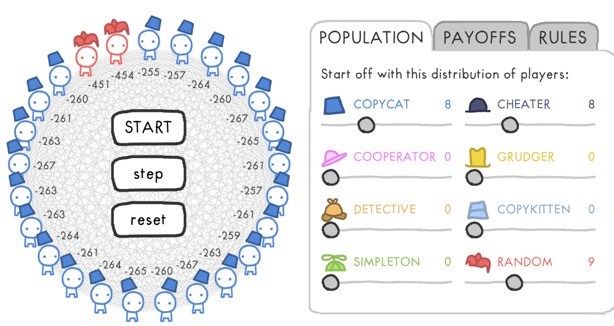

Figure 16 - Prisoner's Dilemma Simulation Elimination 4

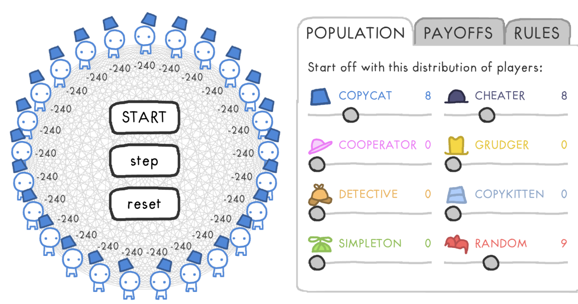

Figure 17 - Prisoner's Dilemma Simulation Elimination 6

Although looking at a one-shot version of the prisoner's dilemma tells us that the cheater should be the dominant one, the other player that had a strategy that adopted the opposing player's strategy won the competition. However, this situation depends on many factors, such as what utility is earned by cooperating/betraying, the number of games played in one round, the strategies of players, and the number of players with a certain strategy profile ("The Evolution").

4. Keynes' Beauty Contest

In a Keynesian Beauty contest, participants are rewarded for choosing the most popular/ chosen candidate. As a result, they do not choose the candidate that they perceive as the most beautiful, but they choose the one that they think the majority thinks is the most beautiful. You could also do that, and picking the face you think is the most attractive would be called the "naive strategy". This approach, of course, isn't only for beauty contests. The reason why it is called a beauty competition is that John Maynard Keynes was inspired by a beauty competition before he wrote his 1936 work *The General Theory of Employment, Interest and Money* .

This theory is commonly used in finance, where investors need to guess the most popular stocks etc. to invest in (The Keynesian Beauty Contest). A game similar to this is "Guessing the ⅔ of the average". The simulation for this contest will be done in the classroom during the presentation. However, this experiment was done before in 2012. The game yielded the following results the first time it was played.

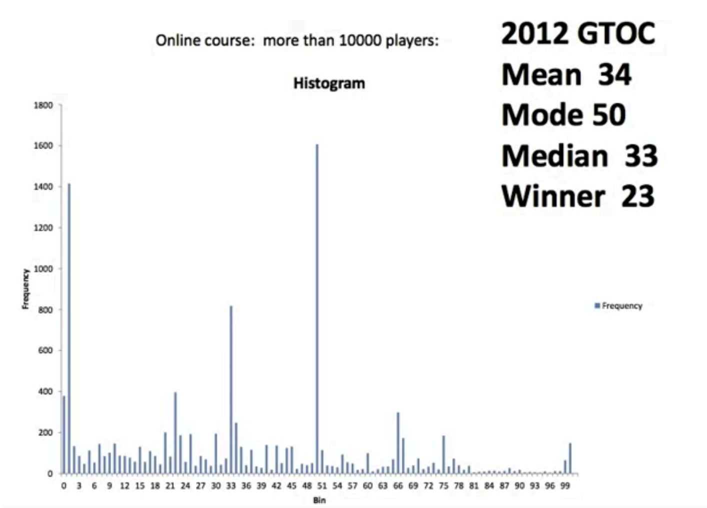

Figure 18 - Keynes' Beauty Contest First Game 2012

According to the assumptions of game theory, one would expect the players to play according to the Nash Equilibrium. However, as you may see from the one-shot game in Figure 18, people were nowhere near the Nash equilibrium in the first round. Normally, you wouldn't expect any players to guess any number above 66.67. Guessing any number that lies above 66.67 is dominated for every player since it cannot possibly be 2/3rds of the average of any guess. However, if this is dominant for every player, every player should be guessing the ⅔ of 66.67 as well which is approximately 44.45. This calculation goes on until the Nash equilibrium approaches 0. Accordingly, in the second round of this game played with the same people, we can see the winner shifting closer to 0 since the players now have a better idea about their guesses ("Game Theory").

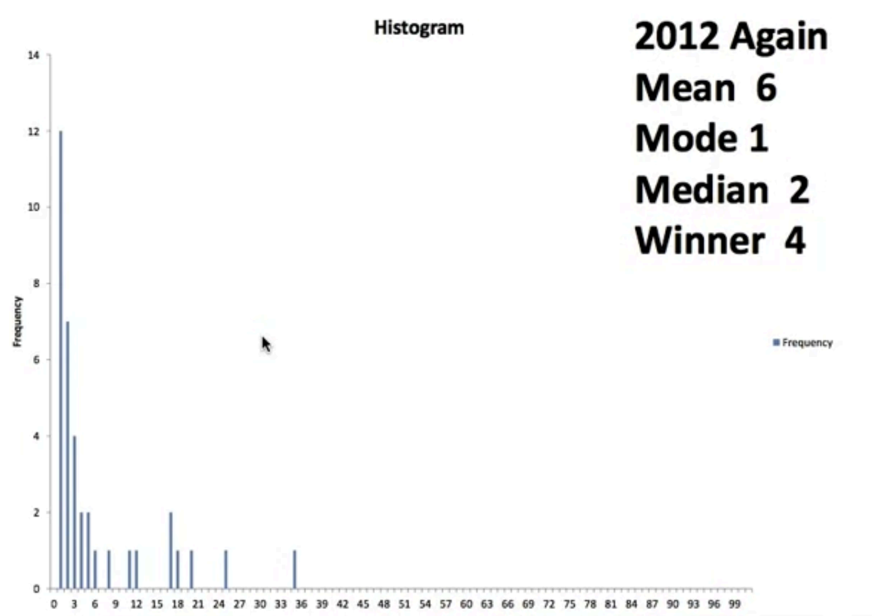

Figure 19 - Keynes' Beauty Contest Second Game 2012

5. Game Theory from a Behavioral Economics Perspective

There is a fact that even the most classical of classical economists can not rebut, people are not rational creatures. However, most of the models of game theory assume that humans are rational and self-interested agents. For this reason, there has been a need to compensate for the errors that the assumptions for games cause. This is how the idea of a behavioral game theory emerged. For example, take the most common and popular game as an example, prisoner's dilemma. Would the prisoners think that betraying one another would be the Nash equilibrium if the players were married or very close friends? Many factors could affect such situations such as relationships, ethics, and other motives of some players ("(Behavioral) Game Theory").

6. Conclusion

Game theory can be considered the science of strategy and choices. It is one of the ways used for analyzing the decisions of an agent to make the most efficient choice possible. It is often used in economics (such as oligopolies), computer science, and many other fields. Game theory involves 2 or more players with interdependent choices and investigates an equilibrium point, Nash equilibrium, in each game. Although the theory is revolutionary and highlights many concepts in terms of decision-making, when it comes to the decision-making of individuals in real life, game theory fails to consider the individuals who behave in a non-rational and non-self-interested manner.

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